B.Sc. (Maths) part I
paper – II
Topic:- The cylinder

Def:- The surface generated by a straight line which moves parallel to a fixed straight line and it always touches a given curve (guiding curve) is called cylinder. The fixed line is called axis of the cylinder and moving line is called generator of the cylinder. The given curve is guiding curve

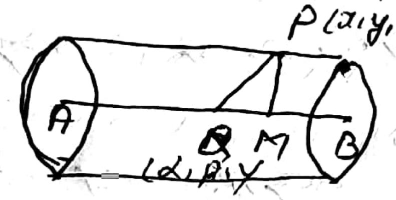
Right cylinder:- Def:- A cylinder in which the fixed line is perp. to the plane of guided curve is called a right cynider

Right circular cylinder
Def. A right cylinder is called a right circular cylinder if the guiding curve is a circle

Theorem :- To find the equation of the right circular cylinder whose radius r and axis is the line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

Soln. –



The eqn. of the axis AB of cylinder are

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

Let $Q(\alpha, \beta, \gamma)$ be any point on AB

The direction ratio of AB $= l, m, n$

The direction cosines of AB

$$= \frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$$

Let $P(x,y,z)$ be any point on the cylinder. Join P and Q. Draw $PM \perp AB$

Then PM = radius of the cylinder $= r$

Now $PQ^2 = (x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2$

PM = projection of PQ on CD

$$= \frac{1}{\sqrt{l^2+m^2+n^2}}\left[(x-\alpha)l + (y-\beta)m + (z-\gamma)n\right]$$

In right angled triangle PMQ we have

$PQ^2 = PM^2 + QM^2$

or $(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2$

$= \frac{1}{l^2+m^2+n^2}[(x-\alpha)l + (y-\beta)m + (z-\gamma)n + r^2]$

This is required eqn of cylinder.

Q:- Find the eqn. of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and guiding the curve is the ellipse $x^2+2y^2=1, z=3$

Soln:- The ellipse is $x^2+2y^2=1$ — (1)

The given line is $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$

The direction ~~cosine~~ ratio of this line are $(1,-2,3)$

Since the generators of cylinder are parallel to the line (1)

∴ The direction ratio of the generator are $(1,-2,3)$

Let $Q(x_1,y_1,z_1)$ be any point on the cylinder

The eqn. of any generator through $Q(x_1,y_1,z_1)$ are

$\frac{x-x_1}{1} = \frac{y-y_1}{-2} = \frac{z-z_1}{3}$ — (3)

If (2) meets the plane $z=3$ at

the point $P(\alpha, \beta, 3)$ then

$$\frac{\alpha - x_1}{1} = \frac{\beta - y_1}{-2} = \frac{3 - z_1}{3} = 1 - \frac{z_1}{3}$$

$\therefore \alpha = x_1 + 1 - \frac{z_1}{3}$ and $\beta = y_1 + \frac{2z_1}{3} - 2$ — (4)

If $P(\alpha, \beta, 3)$ lies on the ellipse (1) then

$\alpha^2 + 2\beta^2 = 1$

or $\left(\frac{3x_1 + 3 - z_1}{3}\right)^2 - 2\left(\frac{3y_1 - 2z_1 - 6}{3}\right)^2 = 1$ {from (4)

or $\frac{1}{9}[9x_1^2 + 18x_1 - 6x_1z_1 + 9 + z_1^2 - 6z_1]$
$+ \frac{2}{9}[9y_1^2 + 4z_1^2 + 36 - 12y_1z_1 - 36y_1 + 24z_1] = 1$

or $9x_1^2 + 18x_1 - 6x_1z_1 + z_1^2 - 6z_1 + 9 + 18y_1^2$
$+ 8z_1^2 + 72 - 24y_1z_1 - 72y_1 + 48z_1 = 9$

or $9x_1^2 + 18y_1^2 + 9z_1^2 - 6x_1z_1 + 24y_1z_1$
$+ 18x_1 - 72y_1 - 54z_1 + 72 = 0$

or $x_1^2 + 2y_1^2 + z_1^2 - \frac{2}{3}x_1z_1 + \frac{8}{3}y_1z_1$
$+ 2x_1 - 8y_1 - 6z_1 + 8 = 0$

Therefore locus of (x_1, y_1, z_1) is

$x^2 + 2y^2 + z^2 - \frac{2}{3}xz + \frac{8}{3}yz$
$+ 2x - 8y - 6z + 8 = 0$